Optimal planning strategy for ambush avoidance

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Problem Description

Autonomous planning in adversarial environment

- Move a vehicle from a given origin $s$ to a desired target set $t$.
- Avoid undesired set of states $Q$.
- Another agent obtains a reward if vehicle in $Q$.
- Recurring journey.
- Structured and unstructured environments.
Problem Description

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- Move a vehicle from a given origin $s$ to a desired target set $t$.
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Isaac “Battleship VS Bomber” Game [1]

- Two Players (BLUE and RED), non-cooperative, zero sum game.
- One dimensional problem

Description

- BLUE wants to go from left to right,
  - picks $y_B$, crossing point.
- RED has a rope of bombs of length $l$,
  - sets one end of the rope at $y_0$.
- BLUE is ambushed if
  $$y_0 \leq y_B \leq y_0 + l.$$
**Related Work**

**Ruckle Game [4, 5]**

- Two Players, non-cooperative, zero sum game.
- Two dimensional environment as lattice.
- Uniform outcome.
- Solution is a mixed strategy.
- Identified sufficient conditions for optimality.

An enactment of a finite ambush game.

The Hunter and Bird Game with $r = 1/12$. 

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Optimal planning strategy for ambush avoidance
Related Work

Joseph & Feron [2, 3]

- Extension to networks.
- Non uniform local outcome.
- Several ambushes.
- More realistic applications.
Related Work

Limitations

- Structured environments only.
- Optimal solution set?
- Local outcome function?
Research Questions

Questions

- Can we exhibit and prove characteristics of the solution?
- How can we tackle unstructured environments?
- Can we characterize the optimal solution set?
- Which factors are important to compute the outcome of an ambush at a given point in the environment?
- Can the local outcome be linked to the vehicle model?
Initial game from Joseph & Feron [2, 3]

**Game**

- Two players, Non cooperative, Zero sum.

**Strategies**

- BLUE (Player 1) chooses a route from origin to destination.
- RED (Player 2) selects a number of locations at which to set up ambushes.

**Environment**

- Network representation \((N, E)\).
- Each node \(j\) is associated with a local outcome \(\alpha_j\).
Example

Figure: Decision flow
Players strategy

- $\{p_{ij}, (i, j) \in E\}$: probability of using an edge in the network,
- Flow constraints: $p_{in} = p_{out}$ at each node,
- $p_{sink} = p_{source} = 1$.

RED’s strategy

- $\{q_j, j \in N\}$: probability of using a node in the network,
- $\sum_{j \in N} q_j = 1$. 
Flow Propagation

Constraints

\[
\begin{align*}
\sum_{i | (i, j) \in E} p_{ij} &= \sum_{k | (j, k) \in E} p_{jk}, \quad \forall j \in N \setminus \{s, t\} \\
\sum_{j | (s, j) \in E} p_{sj} &= 1 \\
\sum_{j | (j, t) \in E} p_{jt} &= 1
\end{align*}
\]

(1)
Player's strategy - BLUE

A strategy for BLUE associated with a subset $P$ of edges is a mapping $p$ from $E$ to $[0, 1]$ such that

$$p : E \rightarrow [0, 1]$$

$$(i, j) \mapsto \begin{cases} 
0 \leq p_{ij} \leq 1, & (i, j) \in P \\
0, & (i, j) \in E \setminus P 
\end{cases}$$

and the flow constraints are satisfied.
Players strategy

Player’s strategy - RED

A strategy for RED reduced to \( Q \) is a mapping \( q \) from \( N \) to \([0, 1]\) such that

\[
q : N \rightarrow [0, 1]
\]

\[
j \mapsto \begin{cases} 
0 \leq q_j \leq 1, & j \in Q \\ 
0, & j \in \mathbb{N} \setminus Q
\end{cases}
\]

and \( \sum_{j \in Q} q_j = 1 \).
Outcome

Local outcome $\alpha_j$

The local outcome at a node $j$ in the network is the reward received by RED if it sets up an ambush at nodes $j$ and BLUE’s path goes through node $j$. It is dependent on the characteristics of the local environment.

Global outcome $V$

At each node $j$, the probability that BLUE gets ambushed is equal to the probability that BLUE’s path goes through $j$ times the probability that RED sets an ambush at this node. The global outcome of the game is the sum of these values over all nodes $j$ in the network, weighted by the local outcome at node $j$.

$$V = \sum_{j \in N} \sum_{i \mid (i,j) \in E} p_{ij} q_j \alpha_j = q^T D p. \quad (2)$$

Note: $D_{jk} = \alpha_j$ if the $k^{th}$ line of $p$ represents the probability that BLUE uses an edge $e_{ij}$ directed towards $j$, and $D_{jk} = 0$ otherwise.
Game Formulation

Game 1

\[
\begin{align*}
\min_{p} \max_{q} & \quad q^t D_p \\
\text{subject to} & \quad A_p = b, \\
& \quad p \geq 0.
\end{align*}
\]
Introduction

a slack variable $z$ constrained as follows.

$$z \geq \sum_{i|(i,j) \in E} p_{ij} \alpha_j, \quad \forall j \in N$$

LP 1

minimize $z$

subject to

$$Dp - 1z \leq 0,$$

$$Ap = b,$$

$$p \geq 0.$$
Problem of LP 1
LP Formulation

LP 2

\[
\text{minimize } (1 - \lambda)z + \lambda E \\
\text{subject to } \quad Dp - 1z \leq 0, \\
E = \sum_{i,j : (i,j) \in \mathcal{E}} p_{ij} \| e_{ij} \|_2, \\
A p = b, \\
p \geq 0, \\
\lambda > 0.
\]  

(6)

Note: \( \lambda \simeq 10^{-3} \times \min_j \alpha_j \)
Example

**Figure**: Decision flow - The network representation of the environment (top left) is considered in order to create a strategy for BLUE (top right): \( \mathbf{p} = (p_{12}, p_{13}, p_{14}, p_{25}, p_{26}, p_{35}, p_{36}, p_{45}, p_{46}, p_{57}, p_{67}) = (2/3, 1/3, 0, 2/3, 0, 0, 1/3, 0, 0, 2/3, 1/3) \). Then a strategy for RED (bottom) is computed: \( \mathbf{q} = (q_1, \ldots, q_7) = (0, 1/2, 0, 0, 1/2, 0, 0) \). The outcome of the game for these strategies is \( V = 1/2 \). Strategies displayed here may not be optimal. The probability associated with each edge is displayed as the width of the edge.
The optimal value of the game for a uniform local outcome $\alpha_u$ is

$$\frac{\alpha_u}{c},$$

where $c$ is the minimal $s$-$t$ cut capacity of the network.

If BLUE has at least $c$ distinct paths with equal probability in its routing strategy, then this strategy is optimal.
Limitations

So far...

- Discrete approach.
- No sensitivity analysis regarding $\lambda$.
- Uniform local outcome only.
Extension
to
unstructured environments
Network construction

What’s new?

- Ambush by zone (not punctual)
- Sampling for network nodes
Network construction

(a) Area size = 6

(b) Area size = 10

Figure: Simple network example - Ambushes by area. The probability associated width each edge is displayed as the width of the edge. The orange rectangles represent the intensity of the local outcome on corresponding ambush areas. The origin and destination area are supposed to have a zero local outcome. Grey edges are unused or used with very low probability. Red circles represent the strategy chosen by RED, i.e., areas $i$ where RED might set his ambush with probability $q_i > 0$. 
# Different construction methods

<table>
<thead>
<tr>
<th>Network #</th>
<th>Sampling</th>
<th>Connectivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1(rdm)</td>
<td>Random</td>
<td>Delaunay triangulation</td>
</tr>
<tr>
<td>2(uni8)</td>
<td>Uniform</td>
<td>8 connected grid</td>
</tr>
<tr>
<td>3(uniD)</td>
<td>Uniform</td>
<td>Delaunay triangulation</td>
</tr>
</tbody>
</table>

**Table**: Different network construction methods.
Figure: Results of the optimization for different network geometries. E. In the left column, cycles can be identified in the departure and arrival areas whereas there are only outgoing edges in the right column examples.
Four metrics are used.

- The *entropy* is high for deceptive routes and close to zero for non-deceptive ones.

- The *spreading* of the route measures the portion of the environment covered by the route as a fraction of the total surface of the environment.

- The *strategic outcome* $V$ accounts for the expected losses for BLUE during one trip.

- The *energy* metric, $E = \sum_{i,j|(i,j)\in\mathcal{E}} p_{ij} \|e_{ij}\|$, represents the divergence of the stochastic strategy from the shortest path between origin and destination.
Sensitivity Analysis

(a) Outcome - $\lambda = 0$

(b) Outcome - $\lambda \neq 0$
Sensitivity Analysis

(c) Spreading - $\lambda = 0$

(d) Spreading - $\lambda \neq 0$
Sensitivity Analysis

(e) Energy - $\lambda = 0$

(f) Energy - $\lambda \neq 0$
Sensitivity Analysis

Figure: Metrics convergence. The metrics are plotted as function of \( \sqrt{N} \). All metrics converge (except the energy metric) for the three methods for both values of \( \lambda \). \textit{rdm} creates slightly less optimal strategies regarding the entropy.
Convergence - Meaning

Figure: Comparison of the results obtained for different network density with the second network construction method. The departure-arrival median is divided in 7 areas, which leads to eight different paths. The routes in (a) and (b) are sensibly alike, illustrating the convergence of our solution when the size of the network increases.
Performances

Converge

BLUE’s strategy converges to a distribution that depends solely on RED’s reach and local outcome.

- The reach of RED in the example is such that the s-t minimal cut has a capacity of 7.
- The optimal theoretical outcome is \( \frac{1}{7} \approx 0.1456 \) leading to an entropy of

\[
\sum_{i=1}^{7} -\frac{1}{7} \log\left(\frac{1}{7}\right) = \log(7) \approx 1.94.
\]
Applications

Purposes for the analysis of the environment

- Complete the existing road network with off-road network to optimize the routing,
- Identify relevant geographical areas for ambushes,
- Compute the local outcome map.

Data sources

- Environment elevation from SRTM (Shuttle Radar Topography Mission),
- Maps from Open Street Map with semantic and usage information.
Risk Factors

Figure: Factors influencing the local outcome computation at a given location.
Agent tasks

Mission types

Based on the task assigned to the agent, the local outcome will not be the same:

- **Scouting mission**: BLUE does not want to be seen at all. Binary local outcome: if BLUE is seen, it looses.
- **Transport mission**: BLUE wants to move goods from A to B. Local outcome is linked to the proportion of goods that are delivered safely.
- **Patrolling mission**: a potential encounter means a win for BLUE.
Example 1: Road Network

Assumption

For the remainder of the presentation: the local outcome $\alpha$ is a parameter that is inversely proportional to the maximum speed of the vehicle.

(a) Speed limit in the city of Monaco.  
(b) Optimal strategy

Figure: Example of solution for the road network of the city of Monaco imported from OSM data. Ambushes are supposed to take place at nodes because the portion of off-road environment is negligible.
Example 2: Unstructured Environment

**Maximum speed**

The maximum speed is a combined function of the topological factors and BLUE’s means of locomotion.

- Pedestrian has about constant speed everywhere,
- Car has lower max speed in hilly areas.

**Figure:** Optimal routing strategy for a pedestrian and a car near Fort Irwin, CA. The pedestrian is slow hence its local outcome is high everywhere. The car’s outcome is low in the lowlands (clear color) because it can go much faster.
Future Work

What now?

- Continuous flow model
- Realistic path generation
- Relation to vehicle model
- Air traffic management parallel
Acknowledgement

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